

Concentration of solar radiation by white backed photovoltaic panels

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In this paper, we present an analysis of the concentration achieved by white backed photovoltaic panels. Concentration is due to the trapping by light scattered in the refractive plate to which the solar cell is bonded. Using the reciprocity relation and assuming the ideal case of a Lambertian distribution, a detailed model is formulated that includes the effects of the thickness and walls of the concentrator. This model converges to the thermodynamic limit and is found to be consistent with experimental results for a wide range of cell sizes. Finally, the model is generalized to multiple-cell photovoltaic panels.

I. Introduction

Much of the interest in photovoltaic power has turned to low concentration arrays. A device which is already used by photovoltaic panel manufacturers involves light trapping via total internal reflection. This concentrator is created inadvertently when the cells are assembled into panels. The solar cells are usually bonded to a glass plate which protects them from damage. Additionally, a white plastic backing sheet is used to protect the cells from moisture damage. Light that is incident on the glass is either absorbed by the solar cell or reflected from the white surface. This reflected light can subsequently be collected by the solar cell if it is totally internally reflected from the glass plate-air interface. The cell thus collects part of the light incident on the white area in addition to that which it collects directly (see Fig. 1). The device has a 180° acceptance angle and can collect a useful amount of diffuse radiation on a cloudy day. Engineering studies have shown that under many conditions this configuration can significantly increase the economic feasibility of solar generated electricity.^{1,2} In a previous paper, we described the dependence of such a system on the plate radius and thickness, optical parameters, and cell dimensions.³ Recently, this model has been extended by other investigators to include

absorption effects in the plate.⁴ The effect of placing the cell on the edge of the plate has also been considered.⁵

During the course of the investigations into the characterization of this collector, questions have arisen as to the nature of the angular distribution of the reflected light and the effects of the walls of the concentrator. In this paper, we shall analyze these effects. We will consider the effect of painting the edges of the plate white. The model described in our earlier paper will be further developed, assuming a Lambertian distribution for the scattered light and applying the principle of reciprocity. Finally, we will generalize the theory and discuss how it may be used to analyze the operation of multiple-cell photovoltaic panels.

II. Theory

A. Background

In this section, by way of review, we consider some important properties of the Lambertian distribution and discuss how deviations from it can arise. The general definition of intensity (in W/sr) is $dI = B dA_x \cos\theta_x$, where B is the brightness or radiance (in W/sr-m²) of the emitting surface x . For a Lambertian surface, such as a piece of paper, the brightness or intensity per unit projected area is a constant and independent of direction, as defined by θ_x , which is measured from the normal of the emitting surface. The intensity is then $I = I_0 \cos\theta$, where $I_0 = \int B dA_x$. This is the governing equation for this distribution. The power on the illuminated surface y due to a small area element dA_x of x is given by

$$dP = I d\Omega = B A_x \cos\theta_x d\Omega \quad (1)$$

(see Fig. 2). Using spherical coordinates $d\Omega = \sin\theta_x d\theta_x d\Phi$ so that the power emitted in any cone of half-angle θ_0 by a small area dA_x is

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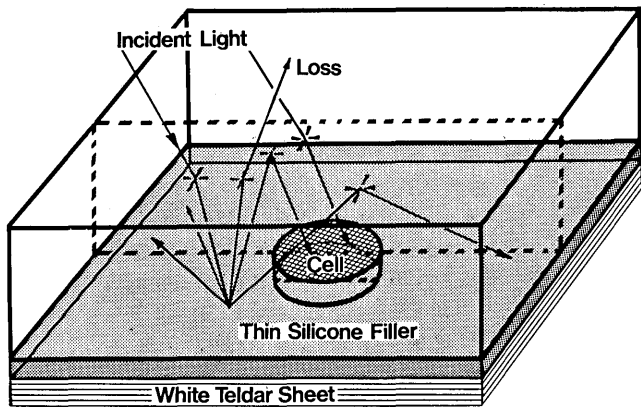


Fig. 1. Typical photovoltaic panel configuration with solar cells bonded to the white backed transparent plate. Light scattered from the white surface may be trapped within the plate and reach the cell.

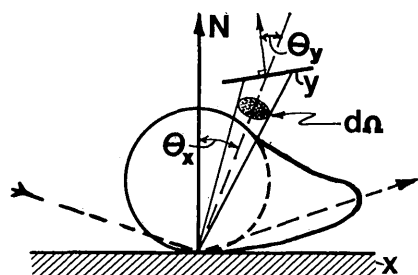


Fig. 2. Circle represents the angular distribution produced by a small area element of a Lambertian surface ($I = I_0 \cos \theta_x$). Specular peak for light incident at 75° illustrates non-Lambertian characteristics of real surfaces. Also shown is the solid angle $d\Omega$ for evaluating the fraction of light emitted from surface x incident on surface y .

$$P = \int I d\Omega = \int_0^{2\pi} \int_0^{\theta_0} I_0 \cos \theta_x \sin \theta_x d\theta_x d\Phi = \pi I_0 \sin^2 \theta_0 \quad (2)$$

Consequently, the total power is πI_0 found by setting $\theta_0 = \pi/2$. We now specifically consider the concentrator illustrated in Fig. 1 and calculate the fraction of radiation that may escape the glass plate. After scattering from the white surface, the rays escaping from the plate lie within a cone of half-angle θ_c , where the critical angle $\theta_c = \sin^{-1}(1/n)$. From Eq. (2), the fraction of light emitted into the loss cone from the bottom is $\sin^2 \theta_c$ or $1/n^2$. This must be multiplied by the transmission T to obtain the fraction lost. In this paper, T will be estimated from the normal incidence transmission into a dielectric⁶

$$T_{12} = \frac{4n_1 n_2}{(n_1 + n_2)^2} \quad (3)$$

where n_1 and n_2 are the refractive indices of the two media.

Figure 2 shows the distribution obtained by shining a He-Ne laser at a white surface for two different incidence angles. At zero incidence angle the distribution is approximately Lambertian. For light incident at an angle of $\sim 75^\circ$, the distribution has a large specular peak. In a white painted plate, the trapped rays will be incident on an element of the white surface at angles $> \theta_c$

and from all directions. Specular reflection would help to produce a more spherical distribution (defined as $I = I_0$ for all θ) because more radiation would be directed at larger angles than predicted by the Lambertian distribution. Previous analyses of the concentration by white painted plates^{3,4} assumed spherical distributions. As we shall see in the next section, this can lead to overestimating the maximum theoretical concentration ratio.

B. Simple Theory

In this section, we will develop two simple models that can be used to understand and describe the concentrator. The first simple approach involves analyzing the light-propagation process as a sequence of individual bounces. This results in an infinite series for the power on the cell P_c as a sum of (1) the energy directly incident on the cell; (2) the energy incident on the cell after once bounce and (3) after two bounces, etc. We will let subscripts i, g, c , and B refer to incident, glass, solar cell, and white bottom area, respectively, and A and T signify area and transmission. The fraction of scattered light which is trapped in the glass plate after each bounce is G . Then, following the path of light as shown in Fig. 3(a), we have

$$P_c = E_i T_{ig} T_{gc} A_c + E_i T_{ig} T_{gc} A_c G + E_i T_{ig} T_{gc} A_c G^2 + \dots = \frac{E_i T_{ig} T_{gc} A_c}{(1 - G)} \quad (4)$$

where we have used the fact that

$$\sum_{n=0}^{\infty} G^n = \frac{1}{(1 - G)}$$

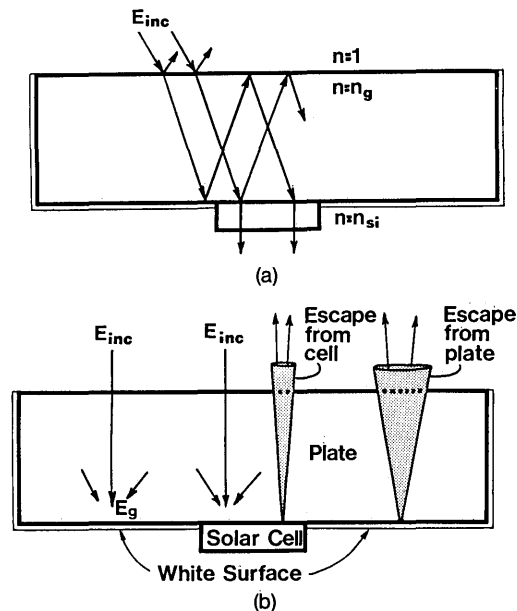


Fig. 3. (a) Geometry for the series approach simple model. Light incident on the glass plate may reach the cell directly after one or more bounces. Note total internal reflection from the top surface. (b) Geometry for the detailed balance simple model. The incident illumination is E_{inc} , and the illumination on the bottom is E_g . Light reflected from the cell and bottom into the critical cone escapes the system.

The power received directly by the cell is $E_i A_c$ times the two transmission factors T_{ig} (air/glass) and T_{gc} (glass/cell). This is the first term in Eq. (4). The remaining terms in the series represent the power into the cell after the light has made one, two, . . . bounces on the white surface.

The fraction G is simply the fraction of the incident light which is reflected from the white surface and cell multiplied by the fraction of this light which is not lost through the critical cone. Hence, if the reflectivity of the white surface is denoted by Ref. we have

$$G = [A_B \text{Ref}/A_g + A_c(1 - T_{gc})/A_g](1 - T_{gi}/n^2). \quad (5)$$

Note that the light reflected off the cell is also Lambertian, and so the same fraction T_{gi}/n^2 will be lost before redistribution on the bottom. The measured concentration C is defined as the power absorbed by a cell in the concentrator P_c divided by the power absorbed by a cell outside the device $E_i T_{ic} A_c$. Therefore,

$$C = P_c/(A_c E_i T_{ic}) = (T_{ig} T_{gc}/T_{ic})/(1 - G). \quad (6)$$

Note that to obtain the ratio of the illumination on the cell inside to outside the device (optical concentration), T_{ic} and T_{gc} are set to unity.¹ If the reflectivity is equal to unity, and the cells are very small ($A_B \approx A_g$), the maximum concentration is

$$C = (T_{ig}/T_{gi})n^2. \quad (7)$$

Thus for the case of small cells, C approaches n^2 , which is the thermodynamic limit for a concentrator with a 180° acceptance angle.^{7,8} So this simple model with a Lambertian distribution and $n = 1.49$ yields $C = 2.2$. However, if one assumes that the distribution is spherical, the fraction of light that can escape is $T_{gc}(1 - \cos\theta_c)$. Replacing T_{gi}/n^2 in Eqs. (6) and (7) by this expression leads to a concentration of 3.86, which is larger than the thermodynamic limit. Since any deviations from the Lambertian distribution must occur so as to preserve the n^2 limit, we see that the spherical distribution previously used³ is not an appropriate approximation to non-Lambertian distributions.

The second simple model follows from that used by Yablonovitch and Cody⁹ to calculate absorption in white backed solar cells. This approach uses the principle of detailed balance to consider the net amount of power leaving or entering a given volume [see Fig. 3(b)]. We will define E_g as the total instantaneous illumination on the bottom surface. To find the concentration we apply conservation of energy. There is one energy balance equation for the glass plate and one for the cell. For the glass plate,

$$E_i T_{ig} A_g = E_g [(1 - \text{Ref})A_B + \text{Ref}A_B T_{gi}/n^2 + A_c T_{cg} + A_c(1 - T_{gc})T_{gi}/n^2], \quad (8)$$

where the left-hand side is the power transmitted into the glass by direct incidence. The terms on the right-hand side are the power lost from the glass by (1) absorption by the white surface, (2) escape from the white surface through the critical cone, (3) propagation into the cell, and (4) reflection from the cell into the critical

cone. Note that we have assumed that due to the high absorption coefficient of the solar cell material a negligible amount of light escapes from the cell. The energy balance for the cell is

$$E_g T_{gc} A_c = P_c. \quad (9)$$

Substituting for E_g from Eq. (8) yields the power P_c absorbed by the cell. The expression one obtains for the concentration is identical to Eq. (6).

Not considered in these simple expressions are the effects of light incident on the walls. Also, both assume that the fractional collection by the cell is not a function of the plate thickness and is equal to the fractional area that it occupies, as was assumed in previous engineering studies.¹ This is not necessarily true. It is for this reason that we now expand the series approach Lambertian model to include the concentrator's dependence on plate thickness and wall reflection. We will then present experimental results to evaluate the validity of our model and to determine where any deviations such as specular reflection have arisen. However, to expand the model we must first introduce reciprocity.

It is a well-known concept in the study of thermal radiation heat transfer¹⁰ that the fractional exchange of energy is related to the areas of the emitters by the reciprocity relation

$$x_y A_y = y_x A_x, \quad (10)$$

where y_x is the fraction of radiation emitted by surface x that is intercepted by surface y . In the notation used in this paper, the subscripted letter indicates the fraction of diffuse light going to a surface, and the subscript indicates the origin of the light. It can be shown that if one restricts the radiation transfer to rays arriving or leaving a surface within a particular angular range smaller than the set of all possible angles, the restricted reciprocity relationship holds:

$$x_y^0 A_y = y_x^0 A_x, \quad (11)$$

where y_x^0 is the fraction of energy emitted by x in a given range of angles arriving at y (see Appendix).

C. Detailed Theory

We now have the tools to expand the series approach Lambertian model to include the effects of the thickness of the plate. The complete process occurring inside the concentrator is shown in Fig. 4. Light reflecting from the white surface has four possible subsequent paths. A fraction l_B is lost through the critical cone. A fraction C_B reaches the cell, and fractions B_B and W_B reach the bottom and wall, respectively. We use subscripts B , W , and C to denote quantities pertaining to the bottom, wall, and cell, respectively [see Fig. 4(a)]. To find the total power involved, we simply multiply each fraction by the area of the white surface and by the incident illumination. We will neglect absorption and transmission losses at present

Considering the light reflected from the white painted walls [Fig. 4(b)], there are also four possible paths. The light can be lost l_W , collected by the cell C_W , incident

on the bottom B_W , or incident on the wall W_W . Thus the two equations that hold for the fractions involved are

$$C_B + l_B + B_B + W_B = 1, \quad (12)$$

$$C_W + l_W + B_W + W_W = 1. \quad (13)$$

To describe completely the concentrator we must determine these eight exchange fractions.

We now consider multiple bounces. After the first bounce, a fraction, Ref, of the incident light is reflected from the bottom, and a fraction, C_B Ref, reaches the cell. After the second bounce, as shown in Fig. 4(b), $C_B B_B$ Ref² + $C_W W_B$ Ref² is collected by the cell. One will note from Fig. 4(c) that the fraction of the incident light reaching a given surface depends on the history of the preceding bounces. A concise manner of expressing the fraction of light reaching a given surface on the $(n + 1)$ th bounce as a function of the n th bounce is the matrix equation:

$$\begin{pmatrix} 0 & \text{Ref}C_W & \text{Ref}C_B & 0 \\ 0 & \text{Ref}W_W & \text{Ref}W_B & 0 \\ 0 & \text{Ref}B_W & \text{Ref}B_B & 0 \\ 0 & \text{Ref}l_W & \text{Ref}l_B & 0 \end{pmatrix} \begin{pmatrix} C \\ W \\ B \\ l \end{pmatrix}_n = \begin{pmatrix} C \\ W \\ B \\ l \end{pmatrix}_{n+1}. \quad (14)$$

Here C , W , B , and l in the column matrices refer to the fractional amount of light at the n th or $n + 1$ st bounce that reaches the cell, wall, or bottom or is lost, respectively. For example, noting that the fractions of the incident light that reach the cell, wall, and bottom on the first bounce are $C_1 = C_B$ Ref, $W_1 = W_B$ Ref, $B_1 = B_B$ Ref, respectively, we obtain for the second bounce the quantity $C_2 = C_B B_B$ Ref² + $C_W W_B$ Ref². If we denote the 4×4 matrix by the symbol M , the sum S_X of the fraction of the incident power on the bottom that subsequently reaches surface X is given by

$$\begin{pmatrix} S_C \\ S_W \\ S_B \\ S_l \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} C \\ W \\ B \\ l \end{pmatrix}_n = (I + M + M^2 + M^3 \dots) \begin{pmatrix} C_B \text{ Ref} \\ W_B \text{ Ref} \\ B_B \text{ Ref} \\ l_B \text{ Ref} \end{pmatrix}, \quad (15)$$

where I is the identity matrix. The last term on the right refers to the first bounce fractions going to each surface as depicted in Fig. 4(a). From matrix algebra it is known that

$$\sum_{n=0}^{\infty} M^n = (I - M)^{-1},$$

so S_C as obtained from Eq. (15) is given by

$$C = 1 + C_B A_B \text{ Ref} / A_C + \frac{C_W A_B [B_B W_B \text{ Ref}^3 + W_B (1 - B_B \text{ Ref}) \text{ Ref}^2] + C_B A_B [W_B B_W \text{ Ref}^3 + B_B (1 - W_W \text{ Ref}) \text{ Ref}^2]}{A_C [(1 - W_W \text{ Ref})(1 - B_B \text{ Ref}) - (W_B B_W \text{ Ref}^2)]}. \quad (17)$$

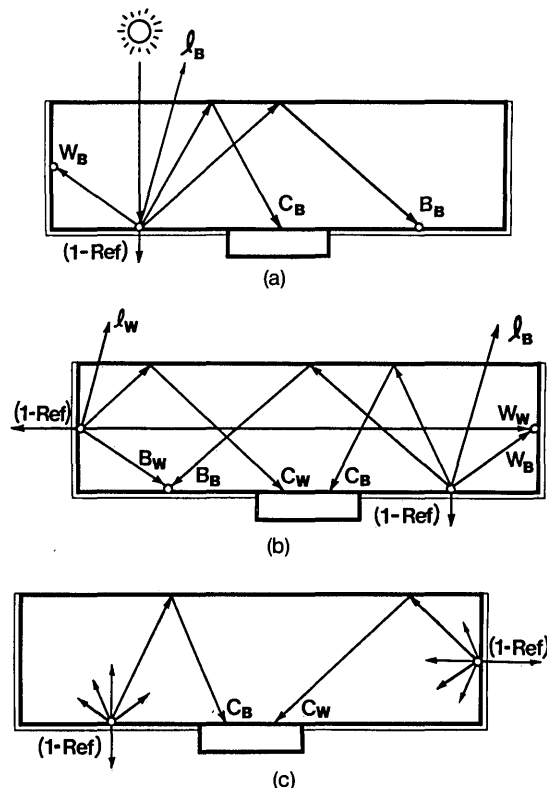


Fig. 4. Path of light through the concentrator showing the history of the light that reaches the cell on the (a) first, (b) second, and (c) third bounces. This is a photon flow diagram. The fraction going to a surface on a given bounce depends on the amount going to each surface on the previous bounce.

$$S_C = \sum_{n=0}^{\infty} C_n = (I - M)^{-1} \begin{pmatrix} C_B \text{ Ref} \\ W_B \text{ Ref} \\ B_B \text{ Ref} \\ l_B \text{ Ref} \end{pmatrix}.$$

Using the definition that the inverse of the $(I - M)$ matrix is its adjoint divided by its determinant, one obtains for S_C

$$S_C = C_B \text{ Ref} + W_B \text{ Ref}^2 \frac{C_W (1 - B_B \text{ Ref}) + C_B B_W \text{ Ref}}{(1 - W_W \text{ Ref})(1 - B_B \text{ Ref}) - W_B B_W \text{ Ref}^2} + B_B \text{ Ref}^2 \frac{C_B (1 - W_W \text{ Ref}) + C_W W_B \text{ Ref}}{(1 - W_W \text{ Ref})(1 - B_B \text{ Ref}) - W_B B_W \text{ Ref}^2}. \quad (16)$$

To find the power on the cell we now multiply each term in the expression for S_C by the area of the reflective part of the bottom A_B and by the incident illumination. We also add to this sum the amount of power going to the cell directly, which for the case of unity transmission factor is $A_C E_i$. The concentration is then given by this summed quantity divided by the power going to a reference cell outside the device (which is also $A_C E_i$). Taking all these conditions into consideration, the concentration by a white painted plate is

In the absence of walls, $W_B = 0$, and this equation is identical to that given in our previous paper.³

If we wish to consider transmission loss at the air-glass interface, we must multiply this expression by T_{ig} as was done in the simple models. If the concentration is measured using a reference cell of nonunity transmission, we divide the given expression by T_{ic} . We cannot consider the effect of the transmission of the cell bonded to the plate simply by multiplying Eq. (17) by T_{gc} , for this would imply that all the light reflected off the cell is lost. This is not the case as some of this light may hit the bottom again. To include this effect rigorously, one must modify the expression given in Eq. (14) to include terms describing the fraction of light reflected by the cell that reach the bottom, cell, wall, etc. However, for the case of diffusely reflected light from a large cell, one may assume that much of the reflected light returns to the cell after a fraction T_{gi}/n^2 is lost from the glass. Thus we multiply Eq. (17) by the effective transmission of the cell, which is approximately $T_{gc} + (1 - T_{gc})(1 - T_{gi}/n^2)$. Here $(1 - T_{gc})$ is the fraction reflected diffusely from the cell.

We now calculate the eight exchange fractions of Eqs. (12) and (13), which are used in the expressions for the concentration ratio. The methods of our previous paper,³ in combination with reciprocity, will be used to calculate these fractions. Throughout the rest of the paper, r is the distance from the center of the plate to an area element on the bottom. The thickness and radius of the plate are denoted by h and R , respectively. The cell diameter is W . We begin by calculating C_W , the fraction of light reflected from the wall which reaches the cell. We use the reciprocity relationship $C_w A_w = W_C A_C$. Therefore, $C_W = W_C (W^2/4)/(2Rh)$. Note that diffuse light does not actually come from the cell, but we use the reciprocity construction to find C_W from W_C , which is easier to calculate. If we approximate the fraction from the whole cell to the wall to be equal to the fraction from the center of the cell to the wall, W_C can be readily calculated as follows:

$$\begin{aligned} W_C &= 1/(I_0\pi) \int I d\Omega \\ &= 1/(I_0\pi) \int_0^{2\pi} \int_{\theta_0}^{\pi/2} I_0 \cos\theta \sin\theta d\theta d\Phi \\ &= (1 - \sin^2\theta'_0) = 4h^2/(4h^2 + R^2), \end{aligned} \quad (18)$$

where θ'_0 is the angle between the normal and the marginal ray that reaches the corner of the bottom via TIR. If $\theta'_0 < \theta_C$, then $W_C = 1 - \sin^2\theta_C$, because $\sin^2\theta_C$ is the fraction lost and all rays not lost will reach the wall.

Although the fraction to the wall does vary with the area element position r , a detailed calculation shows that this variation does not exceed 1%, and hence the approximation is valid. Consequently, we use the expression

$$C_W = (1 - \sin^2\theta'_0)(W^2/4)/2Rh. \quad (19)$$

We now consider the fraction of light from the bottom which hits the total bottom surface a second time, B_B

+ C_B . Note from Fig. 5 that the marginal ray that hits the bottom corner of the wall makes an angle θ'_0 with the surface normal. This angle will be a function of Φ and r . The fraction of light from an infinitesimal area element a distance r from the center of the system reaching the bottom b_B and cell c_B is

$$\begin{aligned} b_B + c_B &= 2/(I_0\pi) \int_{\Phi_{c1}}^{\pi/2} \int_{\theta_c}^{\theta'_0} I_0 \cos\theta \sin\theta d\theta d\Phi \\ &= 1/\pi \int_{\Phi_{c1}}^{\pi/2} (\cos^2\theta_c - \cos^2\theta'_0) d\Phi. \end{aligned} \quad (20)$$

The integral for Φ is taken from Φ_{c1} to $\pi/2$ and not necessarily from $-\pi/2$ because there may be some value of $\Phi = \Phi_{c1}$ so that $\theta'_0 = \theta_c$. At this value of Φ and smaller, there will be no rays to the bottom. The quantity θ'_0 is found from Fig. 5 and the law of cosines as applied to the distance from the scattering point to the wall corner ρ . We obtain

$$\begin{aligned} \cos\theta'_0 &= 2h/(4h^2 + \rho^2)^{1/2} \\ &= 2h/[4h^2 + R^2 + r^2(\sin^2\Phi - \cos^2\Phi) \\ &\quad + 2Rr \sin\Phi(1 - r^2/R^2 \cos^2\Phi)^{1/2}]^{1/2}. \end{aligned} \quad (21)$$

This equation can be solved numerically for Φ_{c1} by setting $\theta'_0 = \theta_c$. If no solution is found, the lower limit in Eq. (20) is taken as $-\pi/2$. Using the value obtained for Φ_{c1} and Eq. (21) for θ'_0 , $B_B + C_B$ can be evaluated by integrating Eq. (20).

The total fraction of light going to the entire bottom is the integral of $b_B + c_B$ over the white bottom area divided by this white area plus the light that can (geometrically) escape but is reflected from the air-plate interface. This yields

$$B_B + C_B = 1/A_B \int_{W/2}^R (b_B + c_B) 2\pi r dr + l_B(1 - T_{gi}). \quad (22)$$

To calculate C_B , the fraction of light going from the white bottom surface to the cell, we modify the equations used to obtain $B_B + C_B$. Given the fraction of light B_C that can geometrically fall on the bottom from the cell, we can use reciprocity to calculate the opposite fraction C_B .

$$C_B = B_C \frac{(W^2/4)}{(R^2 - W^2/4)}. \quad (23)$$

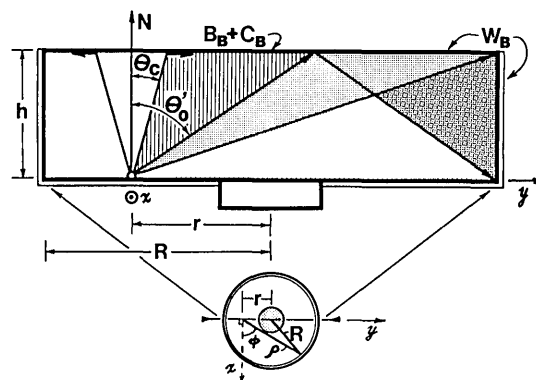


Fig. 5. Geometry for calculation of the fraction of light emitted from the bottom that may reach the bottom, $B_B + C_B$. The top view shows the azimuthal angle Φ .

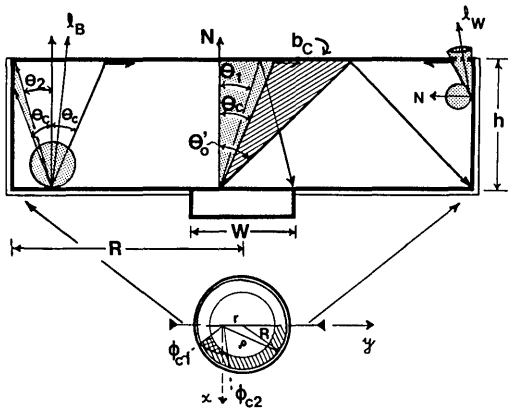


Fig. 6. (Top) Geometry for the calculation of b_c showing the marginal rays θ_1 and θ'_0 . On the left and right is the geometry for loss from the bottom and wall, respectively. In the circular sketch on the bottom the shaded region indicates the region that may receive rays from the cell directly by TIR. Rays within the critical cone which are reflected reach the crosshatched region.

The fraction of light that can go from the cell to the white bottom B_C is given by

$$B_C = \frac{\int_0^{W/2} b_c 2\pi r dr}{\pi W^2/4}, \quad (24)$$

so that C_B is given by

$$C_B = \frac{\int_0^{W/2} b_c 2r dr}{(R^2 - W^2/4)}. \quad (25)$$

The geometrical fraction of radiation that can reach the bottom from an area element a distance r from the center of the cell b_c is given by analogy with Eq. (20) as

$$\begin{aligned} b_c &= 1/\pi \int_{\Phi_{c1}}^{\pi/2} \int_{\theta_1}^{\theta'_0} \cos\theta \sin\theta d\theta d\Phi \\ &= 1/\pi \int_{\Phi_{c1}}^{\pi/2} (\cos^2\theta_1 - \cos^2\theta'_0) d\Phi, \end{aligned} \quad (26)$$

where θ'_0 is the same marginal ray that was used in the $B_B + C_B$ calculation. The angle θ_1 is defined by the ray that reaches the edge of the cell (see center panel, Fig. 6). It is calculated by replacing R with $W/2$ in Eq. (21). If $\theta_1 < \theta_c$, we replace $\cos^2\theta_1$ with $\cos^2\theta_c$ so that only rays that are outside the loss cone will be considered. Because of the possibility that this inequality is met for some value of $\Phi = \Phi_{c2}$, we split the integral into two parts. The area that may receive rays from the cell is shown in the shaded region at the bottom of Fig. 6. Thus b_c is

$$\begin{aligned} b_c &= 1/\pi \left[\int_{\Phi_{c1}}^{\Phi_{c2}} (\cos^2\theta_c - \cos^2\theta'_0) d\Phi \right. \\ &\quad \left. + \int_{\Phi_{c2}}^{\pi/2} (\cos^2\theta_1 - \cos^2\theta'_0) d\Phi \right]. \end{aligned} \quad (27)$$

Note that if $\theta_1 < \theta_c$ at $\Phi = \pi/2$, Φ_{c2} is $\pi/2$, so the second integral is zero.

To accurately calculate b_c we must include Fresnel reflection at the plate-air interface. The area of the bottom that may receive reflected rays within the critical cone is shown in the crosshatched region of Fig. 6. Adding the two integrals that represent this fraction of light to Eq. (27) we obtain

$$\begin{aligned} b_c &= 1/\pi \left[\int_{\Phi_{c2}}^{\pi/2} (\cos^2\theta_1 - \cos^2\theta'_0) d\Phi \right. \\ &\quad + \int_{\Phi_{c1}}^{\Phi_{c2}} (\cos^2\theta_c - \cos^2\theta'_0) d\Phi \\ &\quad + (1 - T_{gi}) \int_{\Phi_{c1}}^{\Phi_{c2}} (\cos^2\theta_1 - \cos^2\theta_c) d\Phi \\ &\quad \left. + (1 - T_{gi}) \int_{-\pi/2}^{\Phi_{c1}} (\cos^2\theta_1 - \cos^2\theta'_0) d\Phi \right]. \end{aligned} \quad (28)$$

Using this value for b_c in Eq. (25) C_B can be obtained.

We may now find W_B from $W_B = 1 - (B_B + C_B + l_B)$. Then B_W can be calculated using the reciprocity equation $A_W B_W = W_B A_B$, again illustrating the usefulness of reciprocity.

Calculation of the fraction lost from the wall l_W involves a slightly different approach than that used to find $B_B + C_B$. The half-cone of rays about the y axis defining the loss from the wall is shown at the extreme right-hand side of Fig. 6. If we were to use the angles θ and Φ to calculate the solid angles involved, the geometry would be complex (see right-hand side, Fig. 7). However, the fraction l_W can be readily calculated if we integrate using the angle β (the azimuthal angle about the y axis) instead of Φ (the azimuthal angle about the z axis) (see left-hand side of Fig. 7). The Lambertian distribution in terms of θ and β is $I = I_0 \cos\theta \cos\beta$. Therefore,

$$\begin{aligned} l_W &= T_{gi}/(\pi I_0) \int_{-\pi/2}^{\pi/2} \int_{(\pi/2-\theta_c)}^{\pi/2} I_0 \cos\theta \cos\beta \cos\theta d\theta d\beta \\ &= T_{gi} [\theta_c - (\sin 2\theta_c)/2]. \end{aligned} \quad (29)$$

The T_{gi} term accounts for the transmission of the interface.

As stated earlier, the simple equation for the losses from the bottom for a Lambertian surface is $l_B = \sin^2\theta_c$. However, this must be corrected for the light within the critical cone that hits the wall (see left-hand side, Fig. 6). For area elements within a distance $h \tan\theta_c$ of the wall, some of the light in the critical cone will strike the wall. The fraction of radiation from the bottom within the loss cone that intercepts the wall is denoted W_B^0 . Thus the corrected loss from the bottom is

$$l'_B = T_{gi} (\sin^2\theta_c - W_B^0). \quad (30)$$

We can calculate W_B^0 from the restricted reciprocity relation

$$W_B^0 = (A_W/A_B) B_W^0 = B_W^0 2Rh/(R^2 - W^2/4). \quad (31)$$

Here B_W^0 is the fraction of radiation leaving the wall so as to be incident on the bottom in the restricted range $0 < \theta < \theta_c$. Due to the symmetry between the top and bottom surfaces of the concentrator, B_W^0 is the same as l_W previously calculated. The corrected loss from the bottom is thus

$$l'_B = T_{gi}(\sin^2\theta_c - 2Rh_l w/(R^2 - W^2/4)). \quad (32)$$

For the case of a small plate or a large cell we may have $R - W/2 < h \tan\theta_c$. Then some of the light calculated from B_W^0 will fall onto the cell, and B_W^0 will not be equal to l_W . One may estimate the actual W_B^0 by again using the equations used for $B_B + C_B$. The modified W_B^0 is calculated from

$$\begin{aligned} W_B^0 &= 2/(I_0\pi) \int_{-\pi/2}^{\Phi_{c3}} \int_{\theta_2}^{\theta_c} I_0 \cos\theta \sin\theta d\theta d\Phi \\ &= 1/\pi \int_{-\pi/2}^{\Phi_{c3}} (\cos^2\theta_2 - \cos^2\theta_c) d\Phi, \end{aligned} \quad (33)$$

where θ_2 is defined by the ray that reaches the top corner of the plate, and Φ_{c3} is the value of Φ when $\theta_2 = \theta_c$ (see left-hand side, Fig. 6). The quantity θ_2 is given by

$$\cos^2\theta_2 = h^2/(\rho^2 + h^2). \quad (34)$$

Since the white area is relatively small for this case, we may consider the value of θ_2 to be a constant and equal to its value at $r = (R + W/2)/2$ in the expression for ρ given in Eq. (21). This neglects the area integration indicated in Eq. (22).

After finding each of the eight terms in Eqs. (12) and (13) we must modify each to consider the effect of a nonzero absorption coefficient α . To approximate the absorption by the plate, one must consider the path of the light through the concentrator. First, the light is

absorbed by the plate even before it reaches the bottom. For normal incidence, the concentration expression must thus be modified to

$$C' = C \exp(-\alpha h). \quad (35)$$

For light reflecting off the bottom, the strongest trapped intensity occurs at about θ_c . Using this ray's path length the absorption corrected C_B and B_B can be approximated by

$$C'_B = C_B \exp(-\alpha 2h/\cos\theta_c), \quad (36)$$

$$B'_B = B_B \exp(-\alpha 2h/\cos\theta_c). \quad (37)$$

For B_W and W_B one considers the path from the midpoint of the wall to the point midway between the edge of the cell and wall. This yields

$$B'_W = B_W \exp[-\alpha[h^2/4 + (R/2 - W/4)^2]^{1/2}], \quad (38)$$

$$W'_B = W_B \exp[-\alpha[h^2/4 + (R/2 - W/4)^2]^{1/2}]. \quad (39)$$

For C_W , we consider the path from the midpoint of the wall to the midpoint of the cell to yield

$$C'_W = C_W \exp[-\alpha(R^2 + h^2/4)^{1/2}]. \quad (40)$$

Finally, to consider the path from the wall to the wall, we take an average angle of $\pi/4$ as measured from the diameter to obtain

$$W'_W = W_W \exp(-\alpha 2R \cos\pi/4). \quad (41)$$

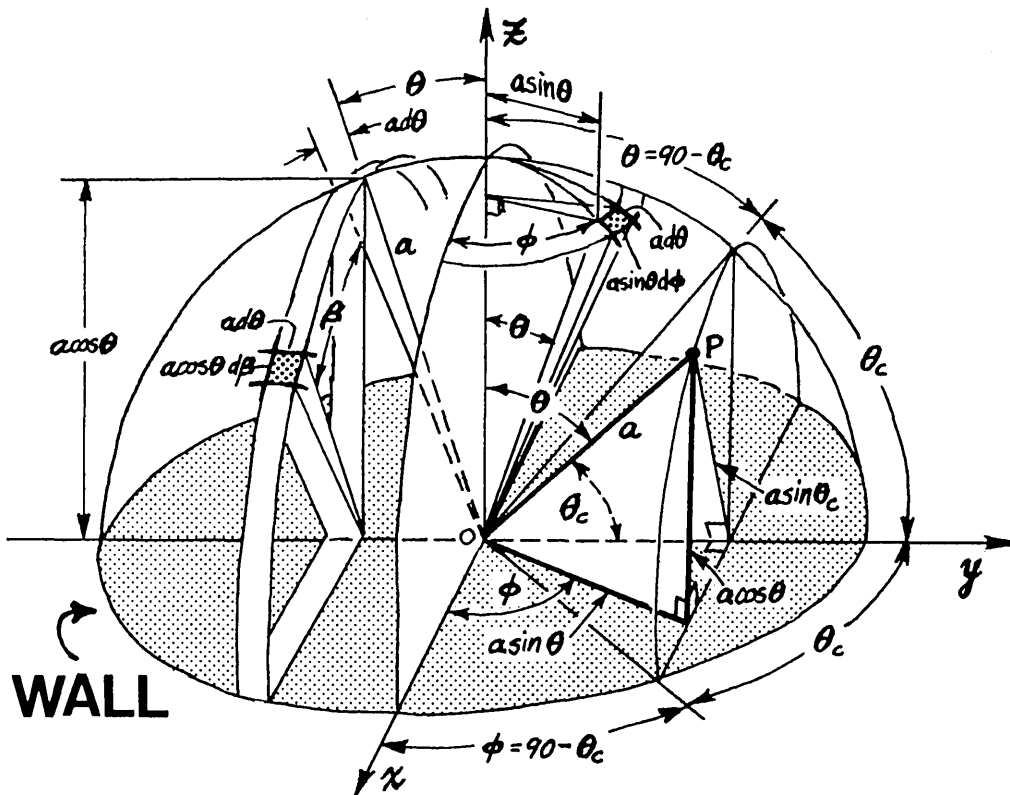


Fig. 7. (Right) Geometry for calculation of the light lost from the wall using θ and Φ . The half-cone of angle θ_c contains the rays that may leave the plate. (Left) Geometry for calculation of l_W using θ and β .

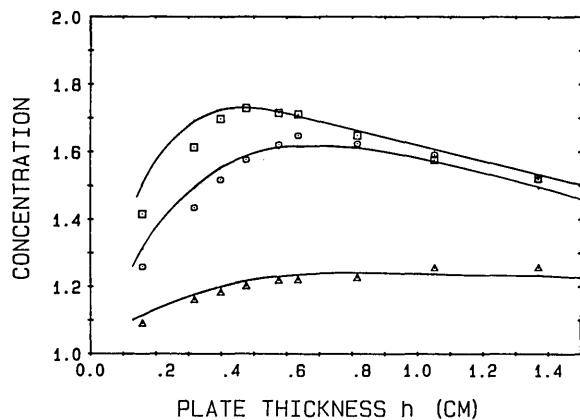


Fig. 8. Experimental and theoretical concentration values as a function of the plate thickness h for three different cell sizes. $W = 1.1, 2.0, 5.08$ cm (squares, circles, triangles). The plate radius R is 3.81 cm. Error in the concentration is ± 0.025 .

Now that we have evaluated the eight terms representing the fraction of scattered radiation going in a particular direction, we can calculate the concentration ratio from Eq. (17). In the next section we compare this theory to measured concentration values.

III. Experimental Materials and Methods

The experimental setup consisted of Spectrolab silicon solar cells bonded with immersion oil ($n = 1.49$) to an unpainted area on the bottom of a round transparent acrylic plate. The bottom of the plate was painted white with the kaolinite clay mixture described in a previous paper.³ To insure constant reflectivity, the thickness of the concentrator was varied by stacking acrylic plates on the first plate with immersion oil between the plates. The edges of the plates were also painted white. Using an integrating sphere, the diffuse reflectivity was found to be 0.82 ± 0.01 at a wavelength of 800 nm (peak response of the Si cell). The absorption coefficient for the acrylic was taken from the manufacturer's data as 0.01 cm^{-1} . To check for the existence of an optical bond between the paint film and the plate, a He-Ne laser was shined onto the plate. A pattern consisting of a bright central spot surrounded by a dark circle, of radius R_r , was then visible if an optical bond was indeed present. This Pfund effect⁶ can also be used to calculate the index of refraction of the plate from $n = (1 + (2h/R_r)^2)^{1/2}$. The concentration ratio was taken as the short circuit current ratio of the cell bonded to the apparatus to that of a cell not bonded to the apparatus but at the same solar orientation. This measurement can be used to obtain the effective concentration since the current is proportional to illumination.

IV. Experimental Results

Figure 8 shows the experimental and theoretical results obtained for the concentrator with $R = 3.81$ cm and $W = 1.1, 2.0,$ and 5.08 cm as a function of the plate thickness h . Figure 9 shows the effect of varying R for $h = 0.318$ cm and $W = 5.08$ cm. Note that theory and experiment agree for a wide range of h and W values.

The correlation does fall off somewhat as the plates become thinner. This could be due to the fact that for small h values the rays that are collected by the cell arrive at very large angles, and, therefore, surface reflection losses are increased. These reflection losses can be accounted for by including the fact that the reflectivity depends on the incidence angle. However, these differences between theory and experiment are small, and the theory describes well all the major features of this concentrator.

V. Generalization to Multiple-Cell Photovoltaic Panels

It is of interest to see how the theory applies to conventional panels where many cells are fixed to a large plate. Considering the area x around one cell in such a panel, the equation relating the exchange of energy between it and the surrounding white area y is the reciprocity relation. [The quantity y_x in Eq. (10) is W_B .] Since the direct incident light on each surface is equal, so is the amount of power emitted from a given area. Therefore, the amount of power leaving an area of radius R around the cell is equal to the power coming into this area from all other areas. In other words, the region can be modeled as an independent concentrator of radius R with no walls. Thus for large photovoltaic panels the concentration can be calculated by the theory with $W_B = 0$.

An important point for designers is that the experimental (and theoretical) maximum in concentration for a large photovoltaic panel will be at a different h value than for a small experimental setup of the same ratio of cell to total area due to the presence of the walls. To model a small system, the walls must be considered.

VI. Conclusion

We have shown that the concentration of a white backed photovoltaic panel is given by Eq. (17). Assuming the distribution of the light to be Lambertian, the thermodynamic limit of n^2 for the concentration is predicted by the theory. Except for thin plates, in which case a greater proportion of the light is incident on the cell at large angles, the theory and experiment correlate well. The deviation is most probably due to the lower solar cell transmission at these angles. Preliminary experiments with textured solar cells show that this surface transmission effect can easily be avoided.

It should be noted that this analysis was made much easier by use of the reciprocity principle. Such an analysis could be applied to more difficult problems such as those being proposed in the field of optical biology. For example, it has recently been demonstrated that etiolated plant tissue exhibits light trapping and scattering similar to the process described in this paper for a photovoltaic concentrator.¹¹ Using the preceding type of analysis, one might gain insight into the operation of the most successful solar converters on earth.

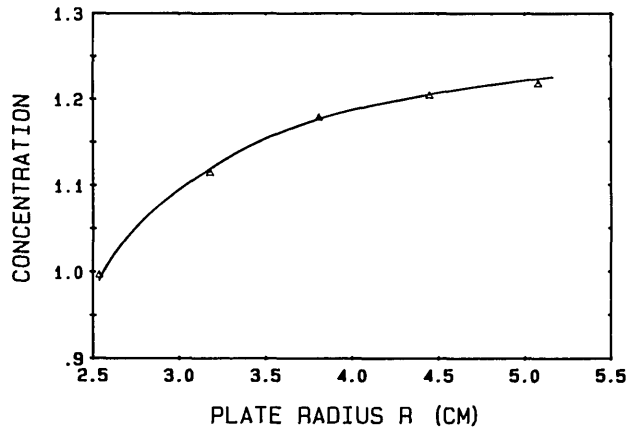


Fig. 9. Experimental (triangles) and theoretical concentration values for $h = 0.318$ cm, $W = 5.08$ cm as a function of R .

Appendix

The concept of reciprocity allows one to calculate the fraction of light from a surface that reaches another surface by knowing the fraction of light that could leave the second surface and reach the first. This is a well-known concept in the study of thermal radiation heat transfer.¹⁰ If we consider the fraction of radiation going from a surface x to a surface y , we find that the exchange factors are given by

$$y_x = \int_{A_x} \int_{A_y} (B_x dA_x \cos\theta_x \cos\theta_y dA_y) / (\pi B_x A_x S^2) = \int (Id\Omega / \text{total power}), \quad (\text{A1})$$

$$x_y = \int_{A_x} \int_{A_y} (B_y dA_y \cos\theta_y \cos\theta_x dA_x) / (\pi B_y A_y S^2), \quad (\text{A2})$$

where S is the distance between the area elements in question. Both θ and S will be a function of the position of the surfaces (see Fig. 2). From these equations we see that $x_y A_y = y_x A_x$. Note that these equations are valid exactly for the Lambertian distribution since the cosine factor appears twice for different reasons. Note also that this result is independent of the fact that some of the radiation could come from total internal reflection. This is true since we can always project reflected rays to a virtual Lambertian source.

Another useful relationship is obtained by restricting the rays that reach each surface to a subset of the total possible exchange. Consider two surfaces x and y that are exchanging diffuse radiation as before. If one considers only those rays leaving dA_x within some restricted range of angles, the integral in Eq. (A1) is carried out over only a portion J of the area A_y , so that JA_y is the region of A_y accessible to rays with the specified angles. To calculate the light from y to x within the same angular range, we can integrate Eq. (A2) over the same restricted area JA_y because of the reversibility of

the paths of emission and incidence. Integrating Eqs. (A1) and (A2) over A_x we can equate the fractions of the radiation reaching each surface in the desired range. We obtain the restricted relation

$$x_y^0 A_y = y_x^0 A_x. \quad (\text{A3})$$

The general and restricted reciprocity relations are powerful tools for the study of light scattering.

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